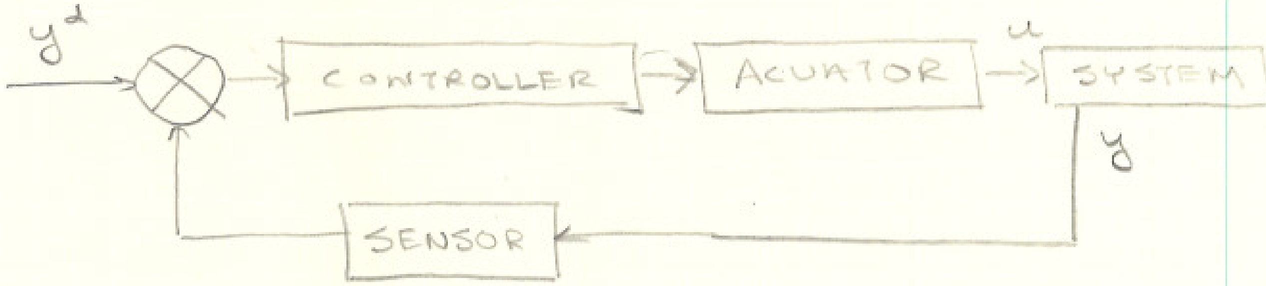
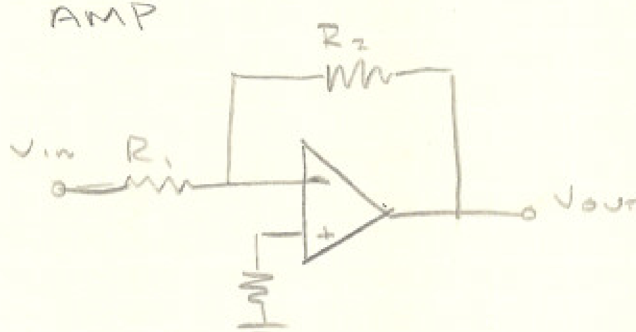
flash.lakeheadu.ca\~tayebi

CLOSED LOOP SYSTEM.



OP AMP



$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

DIFFERENIAL EQU.

- NON-LINEAR DIFFERENTIAL (NOT COVERED IN THIS COURSE)
- LINEAR DIFFERENTIAL

STATE SPACE RERERSENTATION

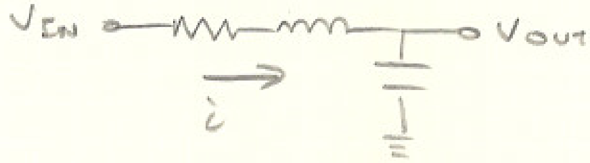
↓

CONTROL DESIGN.

↑

TRANSFER FUNCTION (USING LAPLACE) (ONLY LINEAR)

TRANSFER FUNCTIONS AND BLOCK DIAGRAMS.



$$V_{IN} = Ri + L\frac{di}{dt} + V_{OUT}$$

$$V_{OUT} = \frac{1}{C}\int i\,dt$$

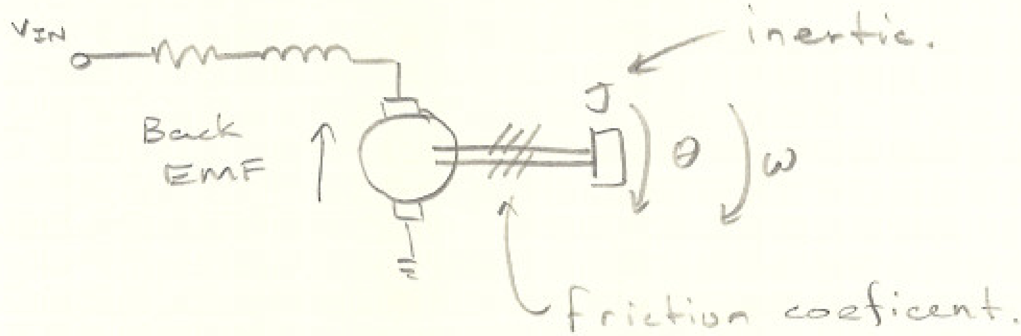
$y = V_{OUT}$

$u = V_{IN}$

$$i = C\frac{dy}{dt}$$

$$u = RC\dot{y} + LC\ddot{y} + y$$

DC MOTOR



electric equ.

$$V_{in} = Ri + L\frac{di}{dt} + E \qquad (A)$$

mech equ.

$$\underbrace{T_m}_{\text{mech torque}} - \underbrace{T_L}_{\text{torque of load}} - \overbrace{B\omega}^{\text{tourqure of friction}} = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt} = (B)$$

ELECTRO MECHANICAL EQUATION.

$$T_m = k_1 i \quad (C)$$
$$E = k_2 \omega \quad (D)$$

equ (D) in (A)

$$V_{in} = Ri + L\frac{di}{dt} + k_2 \omega \quad (E)$$

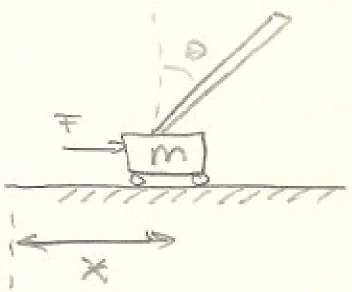
equ (C) in (B)

$$k_1 i - T_L - B\omega = J\frac{d^2\theta}{dt^2} \quad (F)$$

(E) $$\frac{di}{dt} = -\frac{R}{L} i - \frac{K_2}{L}\omega + \frac{1}{L} V_{in}$$

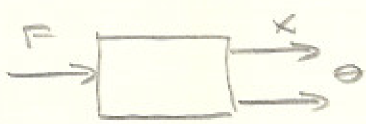
(F) $$\frac{d\omega}{dt} = \frac{K_1}{J} i - \frac{B}{J}\omega - \frac{T_L}{J}$$

EX. PEDULUM ON A CAR.



F: force applied to car
θ: difference of rod.
x: displacement of car
m: mass of the car.

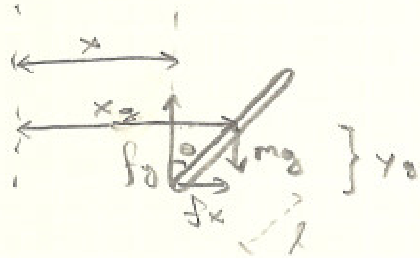
this system has one input an 2 o/p.



FREE BODY DIAGRAM

cart

Rod



$-f_x = Mg$

$f_x = m\ddot{x}_g$

$f_y - mg = m\ddot{y}_g$

$f_x \cos\theta \, l - f_y \sin\theta \, l = I\ddot{\theta}$

$x_g = x + l\sin\theta$

$y_g = l\cos\theta$

mathimatical model for the system.

$$(M+m)\ddot{x} + m\ddot{\theta} l\cos\theta - m\dot{\theta}^2 l\sin\theta = F$$

$$(I + ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta - mgl\sin\theta = 0$$

if $\theta \approx \dot{\theta} \approx 0$, then $\sin\theta \approx \theta$, $\cos\theta \approx 1$

$$(M+m)\ddot{x} + m\ddot{\theta} l = F$$

$$(I + ml^2)\ddot{\theta} + ml\ddot{x} - mgl\theta = 0$$

from last day.

$$Y(s) = \frac{s^2+4s+3}{s(s+1)(s+3)}$$

$$y(t) = ? = \mathcal{L}^{-1}[Y(s)]$$

$$y(t) = \frac{2}{3} + 0.5e^{-t} - \frac{1}{6}e^{-3t}$$

Ex of find value theorem.

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \frac{2}{3}$$

note: that this can only be used when the system is steady state.

Ex of when final value theorem can not be used.

$$y(t) = \mathcal{L}^{-1}\left[\frac{3}{s(s-2)}\right] = -\frac{3}{2} + \frac{3}{2}e^{2t}$$

PARIAL FRACTION EXPRESSION

consider.

$$F(s) = \frac{b_1 s^m + b_2 s^{m-1} + \ldots + b_{m-1}}{s^n + a_1 s^{n-1} + \ldots + a_n}$$

$$F(s) = \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

Assume that P_i are distinct,

$$F(s) = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \dots + \frac{C_n}{s-P_n}$$

$$(s-P_i)F(s) = C_1 + \frac{(s-P_i)C_2}{s-P_2} + \dots \frac{C_n(s-P_1)}{s-P_n}$$

$$\therefore C_1 = \left[(s-P_1)F(s)\right]_{s=P_1}$$

in general,

$$C_i = \left[(s-P_i)F(s)\right]_{s=P_i}$$

for more then one value, (k roots)

$$C_{k-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}\left[(s-P_1)^k F(s)\right]\right]$$

$$i = 0, \dots, k-1$$

EX.

$$F(s) = \frac{s+3}{(s+1)(s+2)^2} = \frac{C_1}{(s+1)} + \frac{C_2}{(s+2)^2} + \frac{C_3}{(s+2)}$$

$$C_2 = \left[\frac{(s+2)^2(s+3)}{(s+2)^2(s+1)}\right]_{s=-2} = -1$$

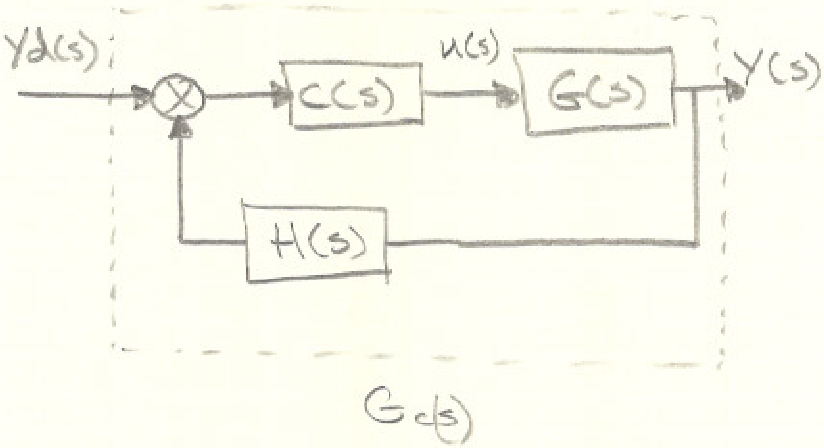
$$C_1 = \left[\frac{(s+3)(s+1)}{(s+1)(s+2)^2}\right]_{s=-1} = 2$$

$$C_3 = \frac{1}{0!}\left[\frac{d}{ds}\left[\frac{(s+2)^2(s+3)}{(s+2)^2(s+1)}\right]\right]$$

$$C_3 = \frac{d}{ds}\left[\frac{s+3}{s+1}\right] = \left[\frac{(s+1)-(s+3)}{(s+1)^2}\right]_{s=-2} = -2$$

$$\therefore \quad F(s) = \frac{2}{(s+1)} - \frac{1}{(s+2)^2} - \frac{2}{(s+2)}$$

BLOCK DIAGRAM.



$$Yd(s) \rightarrow \boxed{G_c(s)} \rightarrow Y(s)$$

$$Y(s) = G(s)\,U(s)$$
$$U(s) = C(s)\left[Yd(s) - H(s)Y(s)\right]$$

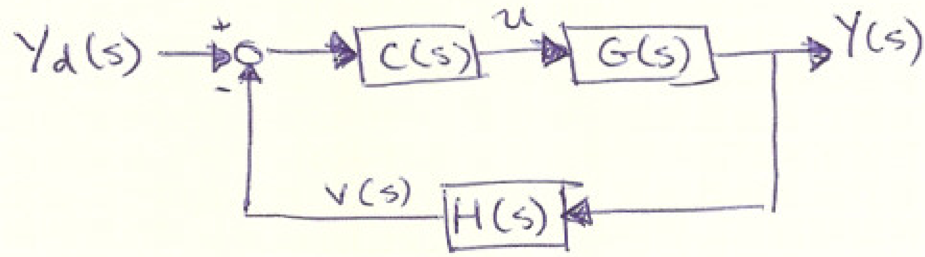
$$Y(s) = G(s)C(s)\left[Yd(s) - H(s)Y(s)\right]$$

$$Y(s) + G(s)C(s)H(s)Y(s) = G(s)C(s)Yd(s)$$

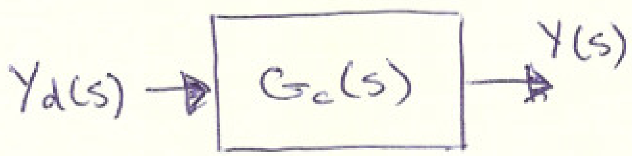
$$\frac{Y(s)}{Yd(s)} = G_c(s) = \frac{G(s)C(s)}{1 + G(s)C(s)H(s)}$$

<u>note</u>: transfer function is equal to the everything in the open path over 1 plus everything.

BLOCK DIAGRAMS

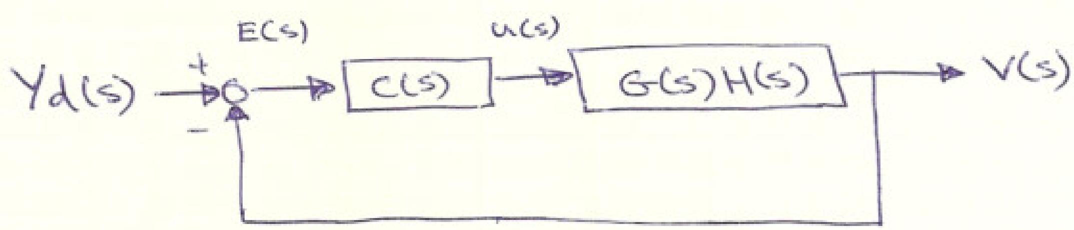


can be simplified to



$$G_c(s) = \frac{Y(s)}{Y_d(s)} = \frac{C(s)\,G(s)}{1 + C(s)G(s)H(s)}$$

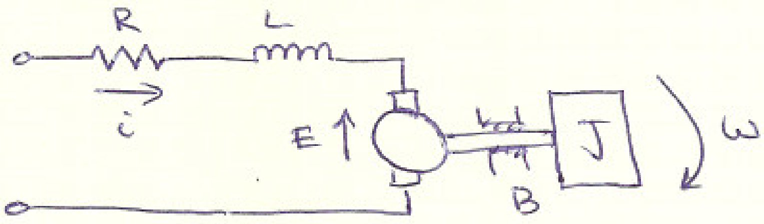
we can change the output we are looking at.



unit feedback

note: input/output are both voltages. Typically in this course $G(s)H(s)$ will be just $G(s)$ with the sensor reading being the output. The sensor is included in $G(s)$.

EX: Armature controlled DC motor.



$$L\frac{di}{dt} + Ri + E = V \qquad (1)$$

$$J\frac{d\omega}{dt} + B\omega = T_m - T_L \qquad (2)$$

$$\frac{d\theta}{dt} = \omega$$

$$E = k_1 \omega$$

$$T_m = k_2 i$$

laplace transform eq (1) & (2)

$$Ls\, I(s) + R I(s) + E(s) = V(s) \qquad (3)$$

$$Js\, \omega(s) + B\,\omega(s) = T_m(s) - T_L(s) \qquad (4)$$

$$s\,\Theta = \omega(s) \qquad (5)$$

$$E(s) = k_1 \omega(s) \qquad (6)$$

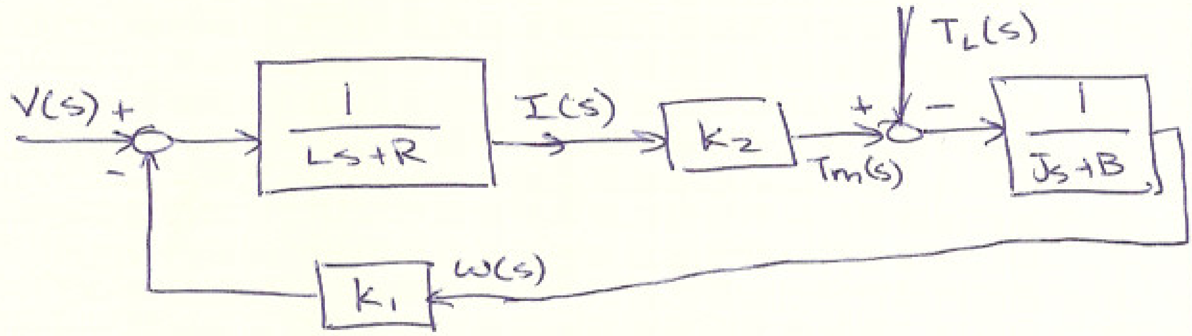
$$T_m(s) = k_2 I(s) \qquad (7)$$

6 in 3 $\Rightarrow (Ls+R)I(s) + k_1\omega(s) = V(s)$ 8

7 in 4 $\Rightarrow (Js+B)\omega(s) = k_2 I(s) - T_L(s)$ 9

8 $\Rightarrow \quad I(s) = \frac{1}{Ls+R}\left[V(s) - k_1\omega(s)\right]$

8 can be represented by the block diagram.



note: you can see that this system is controlled by the voltage in.

in the previous block diagram $T_L(s)$ is known as a disturbance. In order to find the TF we must assume $T_L(s)=0$.

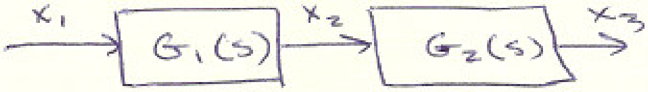
$$\frac{\omega(s)}{V(s)} = G(s) = \frac{\left(\frac{1}{Ls+R}\right)(k_2)\left(\frac{1}{Js+B}\right)}{1+\left(\frac{1}{Ls+R}\right)(k_2)\left(\frac{1}{Js+B}\right)(k_1)}$$

$$\therefore \quad G(s) = \frac{k_2}{(Ls+R)(Js+B) + k_1 k_2}$$

$$\frac{\theta(s)}{V(s)} = \frac{1}{s}G(s)$$

BLOCK DIAGRAM TRANSFORMATIONS

- combining blocks in cascade.

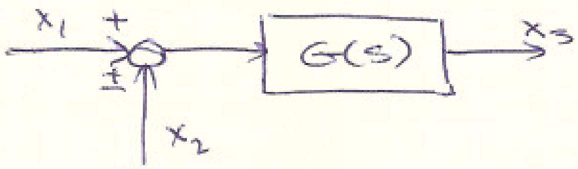


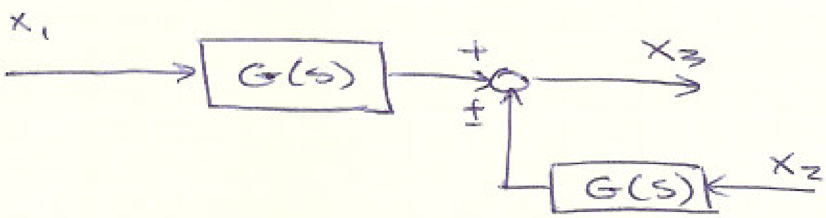
$$x_2 = G_1(s)\,x_1$$

$$x_3 = G_2(s)\,x_2$$

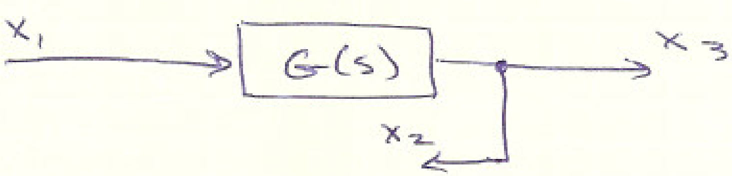
$$\therefore \quad x_3 = G_2(s)\,G_1(s)\,x_1$$

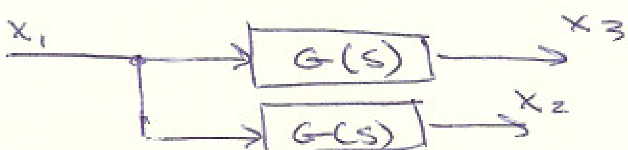
- moving a summing point behind a block.



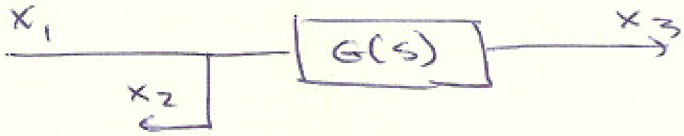


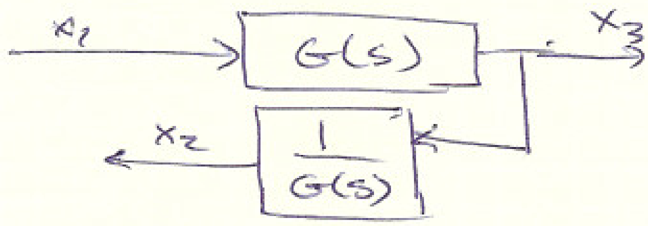
- moving a pickoff point behind a block



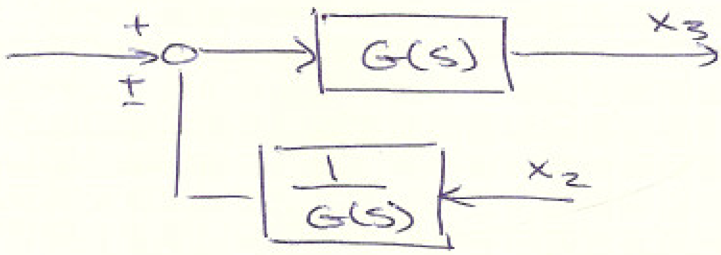


• moving a pick off poin in ~~front~~ behind of a block

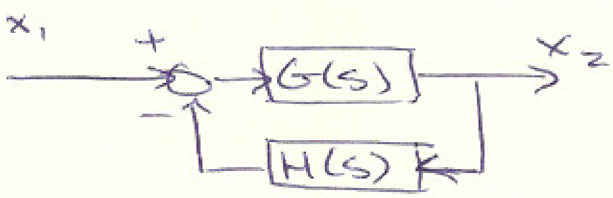


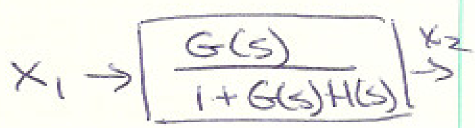


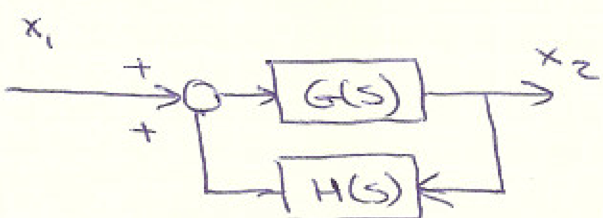
• moving a summing point ahead of a block

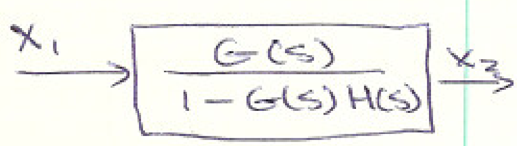


• Eliminatéring feedback.









ZEROS AND POLES OF A TRANSFER FUNCTION.

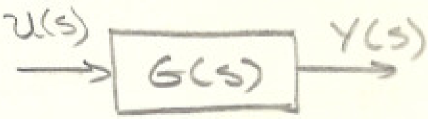
$$G(s) = \frac{N(s)}{D(s)}$$

roots of $D(s)$ are called poles.

roots of $N(s)$ are called zeros.

let P_i be the poles of the system.

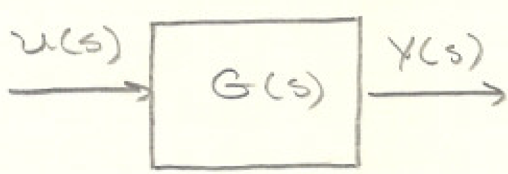
if all the poles are negative real part, then the system is stable.



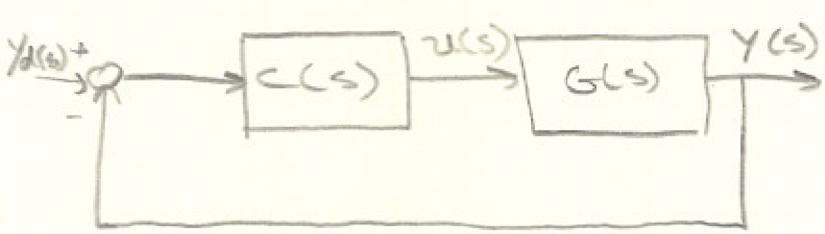
meaning that if we have a bounded input, we have a bounded output.

if only one pole has a positive real part then the system is unstable.

PHYSICAL REALIZATION OF TFs.



b/c this system does not work how we want we add a controlled.



the most common form of industrial controller is the PID.

$$C(s) = K_P + \frac{K_I}{s} + K_D s$$

PRACTICAL PID.

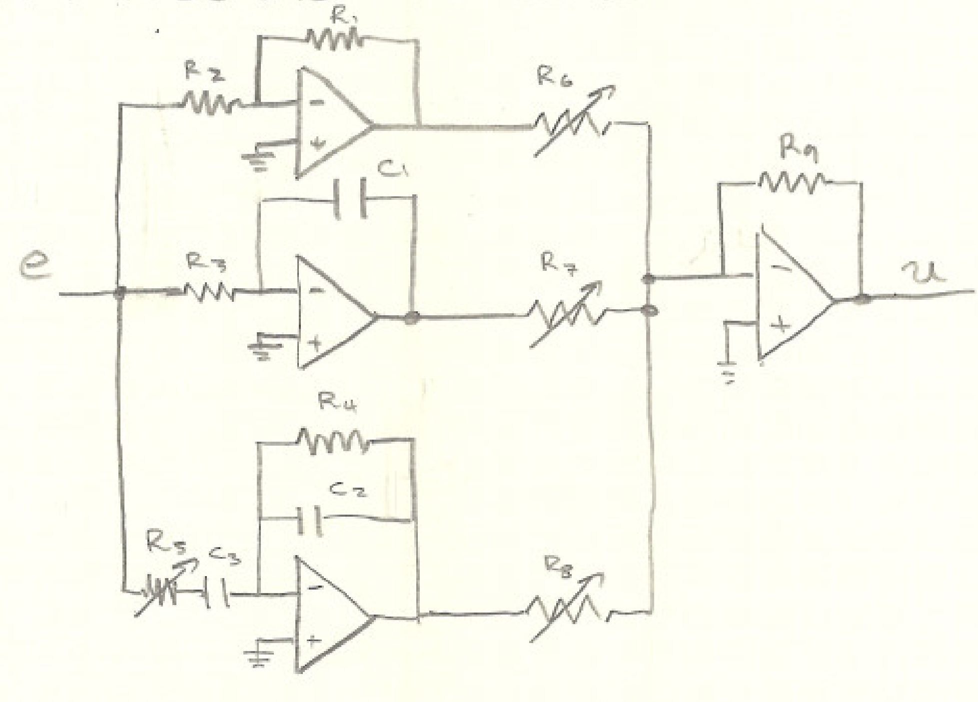
$$C(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{1+\tau s}$$

b/c $K_D s$ amplifies noise, we want to filter it.

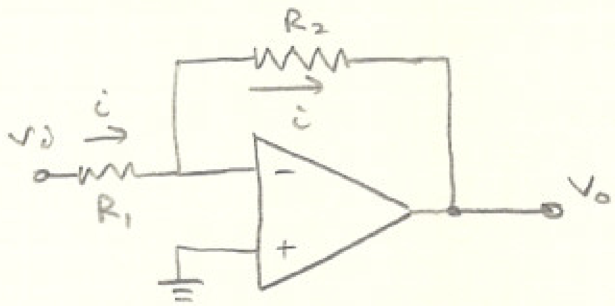
$$\frac{1}{1+\tau s} \rightarrow \text{low pass filter.}$$

where the cutoff freq is $\frac{1}{\tau}$ rad/s.

OPERATIONAL AMPLIFIERS.

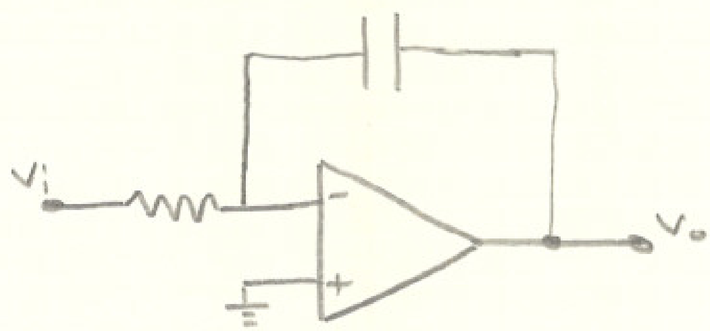


INVERTING AMP



$V_o = \frac{R_2}{R_1} V_i$

INTEGRATING AMP.



① $V_{in} = Ri + \frac{1}{C}\int i\,dt + V_o$

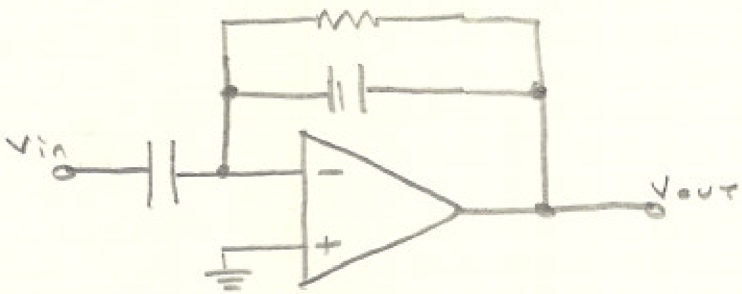
② $V_{in} = Ri$

② in ① $\quad V_o = -\frac{1}{C}\int i\,dt \qquad$ ③

② $\Rightarrow \quad i = \frac{V_{in}}{R} \qquad$ ④

④ in ③ $\quad V_{out} = \frac{-1}{RC}\int_0^t V_{in}\,dt$

DIFFERENTIATING AMP



$$V_{in} = \frac{1}{C}\int i\,dt + Ri + V_{out} \qquad ①$$

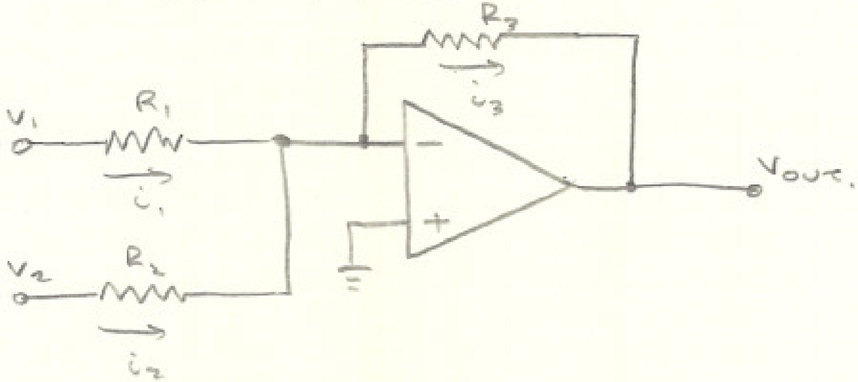
$$V_{in} = \frac{1}{C}\int i\,dt \qquad ②$$

② in ① $\quad V_{out} = -Ri \qquad$ ③

② $\Rightarrow \quad i = C\frac{dV_{in}}{dt} \qquad$ ④

④ in ③ $\quad V_{out} = -RC\frac{dV_{in}}{dt}. \qquad$ ⑤

INVERTING SUMMER



$$V_1 = R_1 i_1 + (i_1 + i_2) R_3 + V_{out} \qquad ①$$

$$V_2 = R_2 i_2 + (i_1 + i_2) R_3 + V_{out}. \qquad ②$$

$$V_1 = R_1 i_1 \qquad ③$$

$$V_2 = R_2 i_2 \qquad ④$$

(3) in (1) $V_{OUT} = -R_3(i_1 + i_2)$ (5)

(3) $\Rightarrow$ $i_1 = \frac{V_1}{R_1}$ (6)

(4) $\Rightarrow$ $i_2 = \frac{V_2}{R_2}$ (7)

(6) and (7) in (5) $V_{OUT} = -R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$ (8)

Let $R = R_1 = R_2$

(8) $\Rightarrow$ $V_{OUT} = -\frac{R_3}{R}(V_1 + V_2)$ (9)

CONTROLS | OCT 02/06 | 1

H/W #1:

Pg 52 # B2_11, B2_13, B2_21, B2_20

Pg 146 # B3_3

Pg 148 # B3_14

Pg 150 # B3_22, B3_

RELISING T.F.

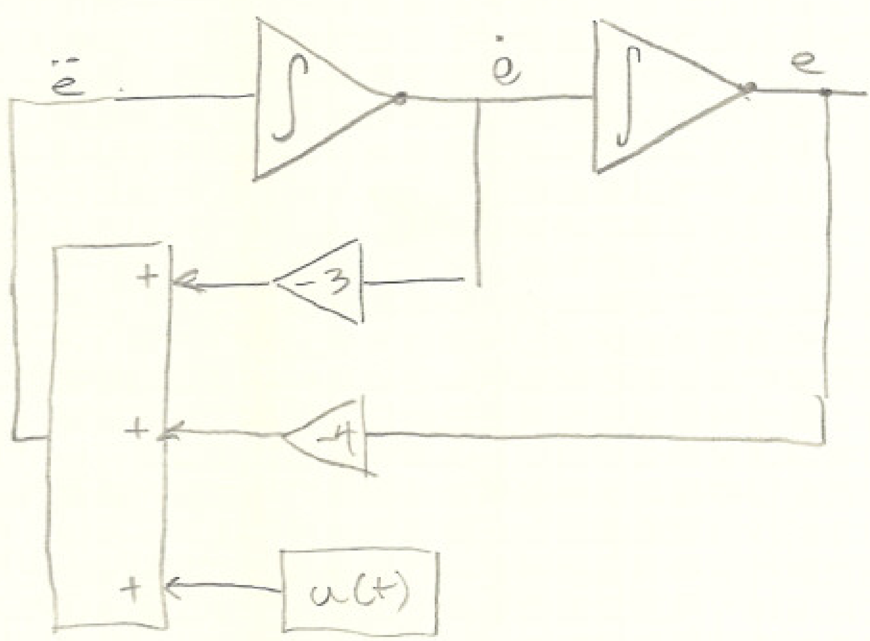
$$C(s) = \frac{1}{s^2 + 3s + 4}$$

$$\frac{E(s)}{U(s)} = C(s)$$

$$s^2 E(s) + 3s E(s) + 4E(s) = U(s)$$

$$\ddot{e} + 3\dot{e} + 4e = u(t) \qquad (1)$$

$$(1) \Rightarrow \ddot{e} = -3\dot{e} - 4e + u(t)$$



STATE SPACE REPERSENTATION.

$$\ddot{y} + a\dot{y} + y = u$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + as + b}$$

$$x_1 = y \Rightarrow \dot{x}_1 = \dot{y} = x^2$$

$$x_2 = \dot{y} \Rightarrow \dot{x}_2 = \ddot{y} = -a\dot{y} - y + u = -ax_2 - x_1 + u$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -ax_2 - x_1 + u$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

IN GENERAL

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \text{state space repersentation.}$$

$\vec{x} \in \mathbb{R}^n$ (state matrix)

$x = \langle x_1, x_2, \ldots x_n \rangle$ are the state variables.

$u \in \mathbb{R}^m$ (i/p vector)

$Y \in \mathbb{R}^n$ (o/p vector)

$A \in \mathbb{R}^{n \times n}$ (state matrix)

$B \in \mathbb{R}^{n \times m}$ (o/p matrix)

$C \in \mathbb{R}^{p \times n}$ (o/p matrix)

$D \in \mathbb{R}^{p \times m}$ (Direct Transmission matrix)

the poles of the system are the eigen values of the state matrix A

$A \in \mathbb{R}^{n \times n}$

$\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ are the eigen values of A if their solution

$$\det(\lambda I - A) = 0$$

EX:

$$\ddot{y} + a\dot{y} + by = u$$

$$T.F. = \frac{1}{s^2 + as + b}$$

$$A = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}$$

$$= |\lambda I - A| = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ b & \lambda + a \end{bmatrix}$$

$$= \lambda^2 + as + b$$

conclude that TF is no unique for finding poles