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KINEMATIC DECOUPLING

PROCEDURE

1) FIND q_1, q_2, q_3 SUCH THAT THE WRIST CENTER p_c IS LOCATED AT

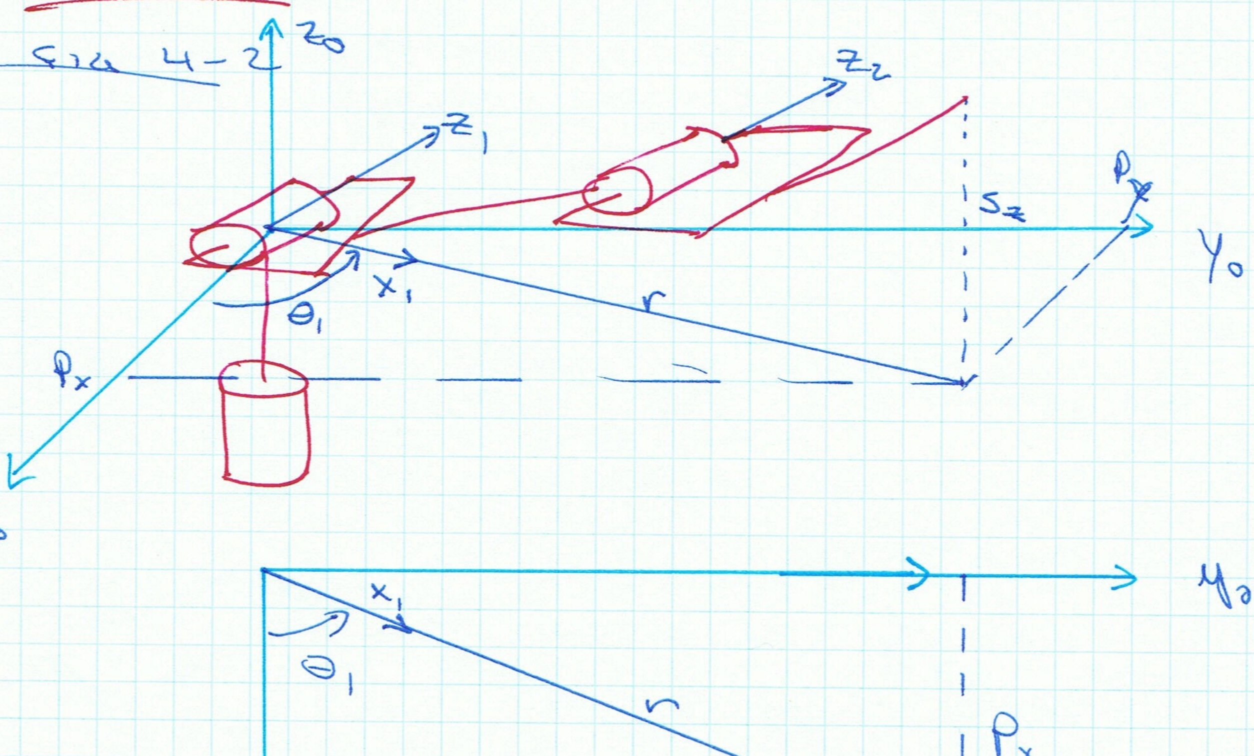
$$p_c = d - R\begin{pmatrix} 0 \\ 0 \\ d_6 \end{pmatrix}$$

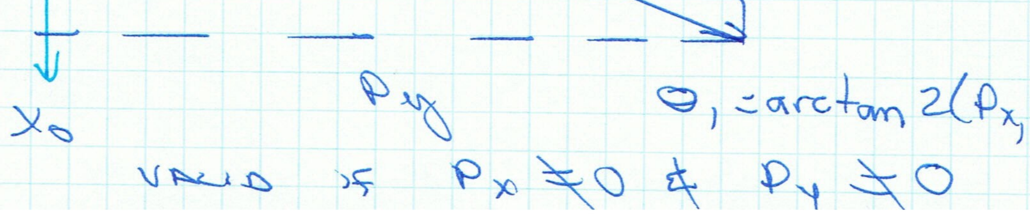
2) USING THE JOINT VARIABLES FOUND IN STEP 1 EVALUATE ${}^0_3R$ (USING FORWARD KINEMATICS)

3) FIND A SET OF EULER ANGLES q_4, q_5, q_6 CORRESPONDING TO ROTATION MATRIX ${}^3_6R = {}^0_3R^T R$

EXAMPLE 1

FIG 4-2





$\theta_1 = \arctan 2(p_x, p_y)$

VALID IF $p_x \neq 0$ & $p_y \neq 0$

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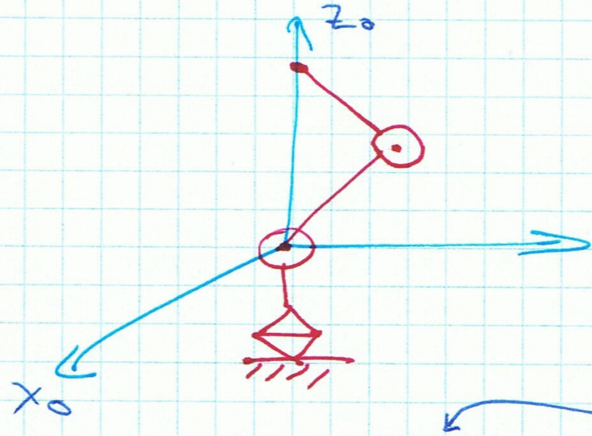
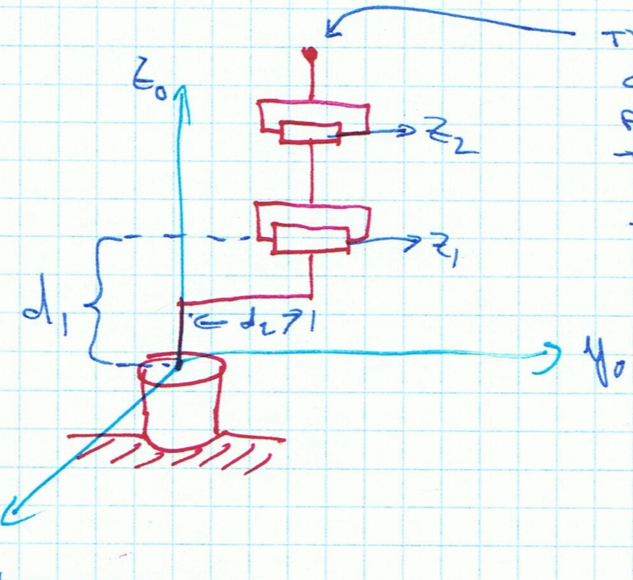
IF $P_x = P_y \Longrightarrow$ SINGULAR CONFIGURATION.
— MEANS WE HAVE INFINITE SOLUTIONS FOR θ_1

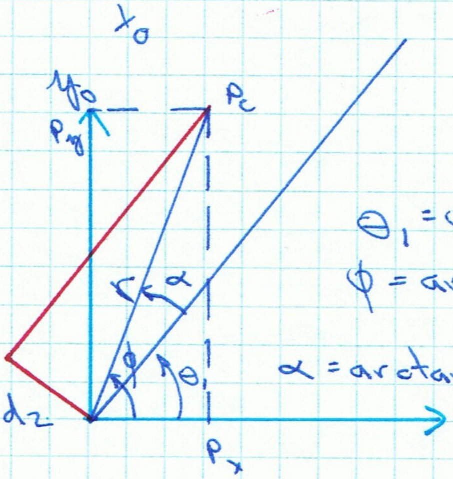
THIS SINGULARITY OCCURS WHEN THE WRIST CENTRE IS ON THE z_0 AXIS



$\Leftarrow$ IF THERE IS AN OFFSET $d_2 \neq 0$



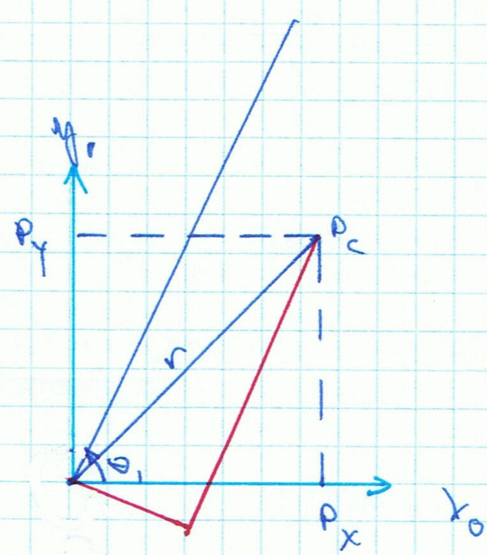
THE WRIST CENTRE CANNOT INTERSECT z_0 AXIS. IN THIS CASE, THERE WILL ONLY BE 2 SOLUTIONS FOR θ_1. THESE CORRESPOND TO THE SO-CALLED LEFT ARM & RIGHT ARM CONFIGURATIONS.



$\theta_1 = \phi - \alpha$

$\phi = \operatorname{arctan2}(P_x, P_y)$

$\alpha = \operatorname{arctan2}(\sqrt{r^2 - d_2^2}, d_2)$



LEFT-ARM

$\longrightarrow \therefore \alpha = \operatorname{arctan2}(\sqrt{P_x^2 + P_y^2 - d_2^2}, d_2)$

RIGHT ARM

$\theta_1 = \operatorname{Atan2}(P_x, P_y) - \operatorname{atan2}(d_2, -\sqrt{P_x^2 + P_y^2 - d_2^2})$

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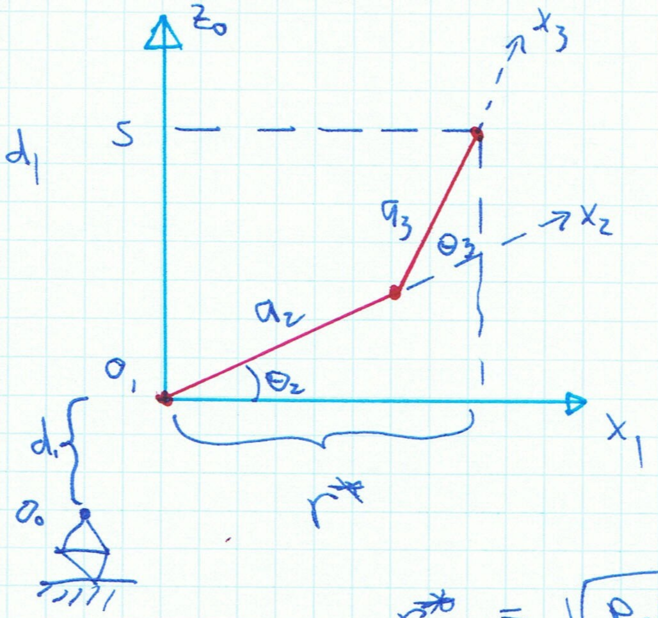
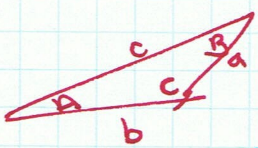
$s = p_z - d_1$



$\theta_2, \theta_3 \leftarrow ?$

$$r^* = \sqrt{p_x^2 + p_y^2 - d_2^2}$$
$$= \sqrt{r^2 - d_2^2}$$

COSINE LAW



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

APPLY THIS TO THIS TO FIND θ_2 & θ_3

$$\cos\theta_3 = \frac{r^{*2} + s^2 - a_2^2 - a_3^2}{2a_2a_3}$$

$$= \frac{p_x^2 + p_y^2 - d_2^2 + (p_z - d_1)^2 - a_2^2 - a_3^2}{2a_2a_3} = D$$

D MUST BE FROM ± 1 $(-1 \le D \le +1)$

$$\theta_3 = \operatorname{atan2}\left(D, \pm\sqrt{1 - D^2}\right)$$

$$\sin\theta_3 = \pm\sqrt{1 - D^2}$$

SIMILARLY WE CAN SOLVE FOR θ_2

$$\theta_2 = \operatorname{atan2}(r^*, s) - \operatorname{atan2}(a_2 + a_3c_3, a_3s_3)$$

THE TWO SOLUTION CORRESPOND TO THE ELBOW UP AND ELBOW - DOWN. POSITIONS.

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ONCE $\theta_1, \theta_2, \theta_3$ ARE OBTAINED WE NEED TO FIND ${}^0_3R$

WE CALCULATE

$${}^3_6R = {}^0_3R^T R = {}^0_6R \leftarrow \text{GIVEN}$$

FOR THE SPHERICAL WRIST WE HAVE:

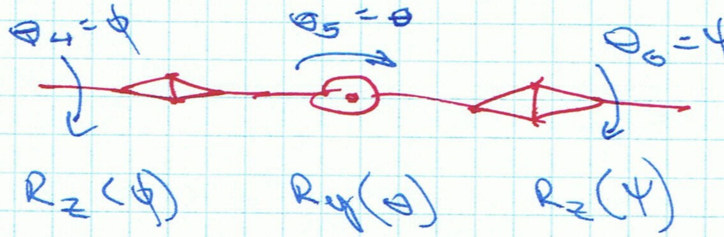
$$\left[\begin{array}{c|c} {}^3_6R & {}^3_6d \\ \hline 0\,0\,0 & 1 \end{array}\right] = A_4 A_5 A_6$$

$${}^3_6R = \begin{bmatrix} C_4C_5C_6 & -C_4C_5C_6 - S_4C_6 & C_4C_5 \\ S_4C_5C_6 & -S_4C_5C_6 + C_4C_6 & S_4S_5 \\ -S_5S_6 & S_5S_6 & C_5 \end{bmatrix}$$

$$= \left.\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}\right\} \text{KNOWN}$$

SOLVE FOR $\theta_4, \theta_5, \theta_6$

CONSIDER SPHERICAL WRIST:



$$\begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi c\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi s\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} = \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}}$$

$${}^3_6R = {}^0_6R^T R \Rightarrow \text{KNOWN}$$

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SUPPOSE THAT NOT BOTH OF u_{13} & u_{23} ARE ZERO THEN $s\theta \neq 0$ AND HENCE BOTH OF u_{31} & u_{32} ARE ZERO

IF NOT BOTH u_{13} & u_{23} ARE ZERO THEN $u_{3} \neq \pm 1$ AND WE HAVE $c\theta = u_{33}$

$$\therefore\ s\theta_3 = \pm\sqrt{1 - u_{33}^2}$$

$$\therefore\ \theta = \text{atan2}\left(u_{33}, \sqrt{1-u_{33}^2}\right)$$

OR

$$\theta = \text{atan2}\left(u_{33}, -\sqrt{1-u_{33}^2}\right)$$

IF WE CHOOSE THE FIRST VALUE FOR θ THEN $\sin\theta > 0$ AND

$$\phi = \text{atan2}\left(u_{13}, u_{23}\right)$$

$$\psi = \text{atan2}\left(-u_{31}, u_{32}\right)$$

EX

$$\frac{s\phi\, s\psi}{-s\phi\, c\psi} = \frac{u_{32}}{u_{31}}$$

$$\text{TAN}\ \psi = \frac{-u_{32}}{u_{31}}$$